Islamic University of Gaza
Faculty of Engineering
Electrical & Computer Dept.
Instructor : Dr. Mohammed Hussein

Date: 28/5/2006
Time: 120 Minutes.

Digital Control
EELE 4471
Final Exam

Solution

Student Name: ........................................ ID Number: ..................

Attempt All Questions.

Question 1 (25 Points)

A) Consider the following discrete system:

$$x(k) \longrightarrow \boxed{h(k)=\frac{1}{2}\delta(k)+\delta(k-1)+\frac{1}{2}\delta(k-2)} \longrightarrow y(k)$$

Find the steady state response of the system to the input $x(k)=5\cos(\frac{\pi k}{4})$.

$$H(z) = \frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}$$

$$\Rightarrow H(e^{j\omega}) = \frac{1}{2} + e^{-j\omega} + \frac{1}{2} e^{-2j\omega} = e^{-j\omega}\left(\frac{1}{2}e^{j\omega} + 1 + \frac{1}{2}e^{-j\omega}\right)$$

$$= e^{-j\omega}\left[1 + \cos\omega\right].$$

$$\Rightarrow |H(e^{\pm j\omega})| = (1 + \cos\omega).$$

$$\angle H(e^{j\omega}) = -\omega.$$

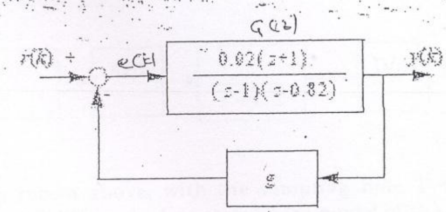
$$\Rightarrow y_{ss}(k) = 5(1+\cos\omega) \cdot \cos\left(\frac{\pi k}{4} - \omega\right).$$

$$\Rightarrow \omega = \frac{\pi}{4}.$$

$$\Rightarrow y_{ss}(k) = 5\left(1+\cos\left(\frac{\pi}{4}\right)\right) \times \cos\left(\frac{\pi k}{4} - \frac{\pi}{4}\right).$$

$$= 8.535 \cos\left(\frac{\pi}{4}(k-1)\right).$$

Consider the following discrete system:




If $g$=1.2, calculate the steady-state error again for the input $r$ given as a *unit step* and a *unit ramp* respectively.

$$e(k) = r(k) - G(z).e(k).g$$

$$\Rightarrow e(k)\left[1 + G(z)\cdot g\right] = r(k)$$

$$\Rightarrow e(k) = \frac{r(k)}{1+g\,G(z)}$$

system type 1.

* unit step $\frac{z}{z-1}$.

$\Rightarrow$ ess

$$e_{ss} = \lim_{z\to 1} \cancel{(z-1)} \cdot \frac{z}{\cancel{(z-1)}} \cdot \frac{1}{1+1.2 \cdot \frac{0.02(z+1)}{(z-1)(z-0.82)}}$$

$$= \frac{1}{1+\infty} = 0$$

* ramp

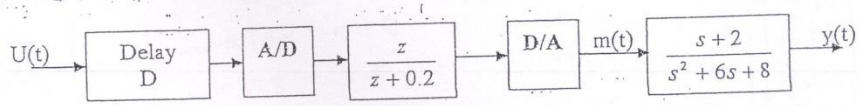
$$\Rightarrow e_{ss} = \lim_{z\to 1} \frac{\cancel{(z-1)}(z)}{(z-1)^{\cancel{2}}} \cdot \frac{1}{1+1.2 * \frac{0.02(z+1)}{(z-1)(z-0.82)}}$$

$$= \frac{z}{(z-1) + \frac{(1.2)(0.02)(z+1)}{(z-0.82)}} = \underline{3.75}$$



=

## Question 2 (25 Points)




Consider the system shown above, with the sampling time T=0.5 sec. Assume that the delay D=2 sec. Find the ZOH equivalent state space model of the plant.

for plant: $T=0.5$

$$A = \begin{bmatrix} -6 & 1 \\ -8 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C = [1 \quad 0], \quad d = 0$$

eign values $\{-2, -4\}$.

$$\Rightarrow e^{AT} = A * \alpha_1 + \alpha_2 I.$$

$$e^{-2T} = \alpha_1(-2) + \alpha_2, \quad e^{-4T} = \alpha_1(-4) + \alpha_2.$$

$$\Rightarrow \alpha_1 = \frac{e^{-2T} - e^{-4T}}{2} = 0.1163, \quad \alpha_2 = 2e^{-2T} - e^{-4T} = 0.6$$

$$\Rightarrow \Phi = e^{AT} = \alpha_1 A + \alpha_2 I = \begin{bmatrix} -0.697 & 0.116 \\ -0.928 & 0.6 \end{bmatrix}$$

$$\Gamma = \int_0^T e^{A\tau} b \, d\tau = \int_0^{0.5} \begin{bmatrix} e^{-4\tau} \\ 2e^{-4\tau} \end{bmatrix} d\tau = \begin{bmatrix} 0.21616 \\ 0.4323 \end{bmatrix}.$$

$$* D = qT + \psi = 3T + 0.5$$

$$\Gamma_1 = \int_{0.5-0.5}^{0.5} e^{A\tau} \, d\tau = \Gamma = \begin{bmatrix} 0.2616 \\ 0.4323 \end{bmatrix}.$$

$$\Gamma_0 = \int_0^{0.5-0.5} e^{A\tau} d\tau = 0$$

$$
\begin{bmatrix} x[k+1] \\ z_1[k+1] \\ z_2[k+1] \\ z_3[k+1] \\ z_4[k+1] \end{bmatrix}
=
\begin{bmatrix}
-0.096 & 0.116 & 0 & 0.2161 & 0 & 0 \\
-0.928 & 0.6 & 0 & 0.4323 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} x[k] \\ z_1[k] \\ z_2[k] \\ z_3[k] \\ z_4[k] \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mu(k) .
$$

Question 3 (25 Points)

A system with a state space description :

$$x[k+1]=\begin{bmatrix}1 & 0.0952\\0 & 0.9048\end{bmatrix}x[k]+\begin{bmatrix}0.0048\\0.0952\end{bmatrix}u[k]$$

This system is obtained by discretizing analog system by a ZOH method using a sampling time 0.1 sec.

a) Calculate a feedback vector L for digital regulator that has a settling time Ts=1.5sec.Use 2-nd order Bessel polynomial?

Hint (2-nd order Bessel polynomial: -4.0530 ± j2.3400)

b) Find the state space description of the digital controlled system?

[a] 1) $W_c = [\Gamma \quad \Phi\Gamma] = \begin{bmatrix} 0.0042 & 0.01386 \\ 0.0952 & 0.08614 \end{bmatrix}$.

$\Rightarrow$ System Controllable

[2] desired Analog roots $\left(\frac{-4.053 \mp j\,2.34}{1.5}\right)$

$= -2.702 \mp j1.56$ .

$\Rightarrow z_{1,2} = e^{s_{1,2}T} = 0.7539 \mp j\,0.11858$ .

$\Rightarrow P(z) = (z - z_1)(z - z_2)$

$= z^2 - 1.5078\,z + 0.5824$ .

[3] $P(\Phi) = \Phi^2 - 1.5078\,\Phi + 0.5824 I$ .

$= \begin{bmatrix} 0.0754 & 0.03779 \\ 0. & 0.0375 \end{bmatrix}$ .

$\Rightarrow L = [0 \quad 1]\, W_c^{-1} \cdot P(\Phi)$ .

$= [7.837 \quad 3.77]$ .

$$\Phi_{new} = \Phi - \Gamma L = \begin{bmatrix} 0.9624 & 0.07709 \\ -0.746 & 0.5455 \end{bmatrix}.$$

$$\Gamma = \begin{bmatrix} 0.0048 \\ 0.0952 \end{bmatrix}.$$

$$C = L = [\,7.837 \quad 3.775\,].$$

$$d = 0.$$

Question 4 (25Points)

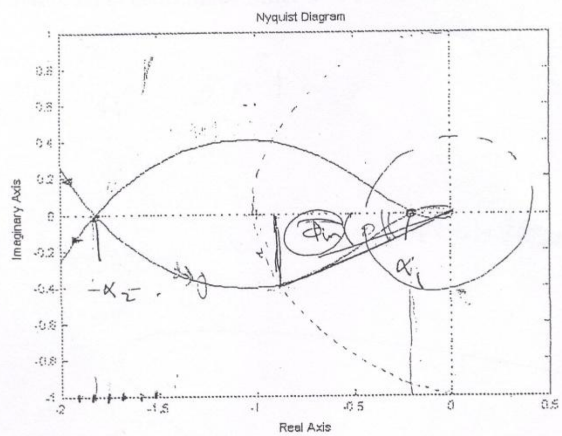
A) Consider the loop transfer function:

$$L(z) = -0.0091\frac{z^3 - 5.4606z^2 + 7.7620z - 3.3035}{z^4 - 3.7811z^3 + 5.3405z + 0.7788}$$

A portion of the Nyquist plot and bode plot are shown below.

a. Define phase margin and gain margin.

b. Find on the graph phase margin and gain margin.




$t\theta =$

$\sin\theta = \frac{0.4}{1}$

23.5

$-0.2$

$\frac{1}{\alpha} = +0.2$

$\alpha = \frac{1}{0.2}$

Gain margin: is the amount of gain in (dB) that can be added to the ~~system~~ loop before the closed loop system becoms unstable

Phase margin:

the amount of pure phase delay can be added to the loop before it becons unstable

$\frac{1}{\alpha} = 1.7$ $\alpha = \frac{1}{1.7}$

$20 \log\left(\frac{-1}{\alpha_2}\right) < G_m < 20 \log\left(\frac{-1}{\alpha_1}\right)$.

$-4.6 < GM < 13.979$

ph

PM = 23.5

The system:

$$x(k+1)=\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x(k)+\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} u(k)$$

$$y(k)=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(k)$$

was obtained by sampling a continuous-time system with a sampling time T=0.1.
Design a state feedback so that the closed-loop system has the following characteristic polynomial in continuous time: $s^3+s^2+s+1=0$.

$W_c = [\Gamma \quad \Phi\Gamma \quad \Phi^2\Gamma]$.

system non controllabile